$Fatou's\ Lemma$:
If $< f_n >$ is a sequence of nonnegative measurable functions and $f_n(x) \to f(x)$ a.e. on a set E, then:

$$\int_E f \leq \liminf \int_E f_n$$

Proof. If A is a set of measure zero with $A \subset E$, then $\int_A f = 0$, thus we may assume without loss of generality that $f_n \to f$ pointwise. Let h be a bounded measureable function which is not greater than f and is identically 0 outside of a set E' of finite measure. Define: $h_n(x) = min\{h(x)\ , f_n(x)\}$, then h_n is bounded by the bound for h and vanishes outside E'. Now $h_n(x) \to h(x)$ for each $x \in E'$ and therefore by the bounded convergence theorem we have:

$$\int_E h = \int_{E'} h = \lim \int_{E'} h_n \leq \liminf \int_E f_n$$

Then taking the supremum over h, we have:

$$\int_E f \leq \liminf \int_E f_n$$

□

$Bounded\ Convergence\ Theorem$:
Let $< f_n >$ be a sequence of measureable functions defined on a set E of finite measure, and assume that there exists $M \in R$ such that $|f_n(x)| \leq M$ for all $n \in N$ and for all $x \in R$. If $f(x) = \lim f_n(x)$ for each $x \in E$, then:

$$\int_E f = \lim \int_E f_n$$