Theorem:
Let f be a non negative integrable function, then the distribution function $F(x) = \int_{-\infty}^{x} f$ is continuous.

Proof. To begin, define $< F_n >_{n=1}^{\infty}$ and $< G_n >_{n=1}^{\infty}$ as:

$$F_n(x) = \int_{-\infty}^{x-\frac{1}{n}} f \;\; and \;\; G_n(x) = \int_{-\infty}^{x+\frac{1}{n}} f$$

then, clearly $< F_n >_{n=1}^{\infty}$ is monotone increasing and it follows that $F_n \to \int_{-\infty}^{x} f$. Next, the sequence $< G_n >_{n=1}^{\infty}$ is monotone decreasing and bounded below by $\int_{-\infty}^{x} f$. Then given this and because $\int_{\mathbb{R}} f < \infty$ by assumption, we know that $G_n \to \int_{-\infty}^{x} f$. Letting $\epsilon > 0$ be given, we know that $\lim_{n\to\infty} F_n = F$ implies that there exists N_1 such that for all $n \geq N_1$ $|F(p) - F_n(p)| = |\int_{-\infty}^{p} f - \int_{\infty}^{p-\frac{1}{n}} f| < \epsilon$. In addition, we know that $\lim_{n\to\infty} G_n = F$ implies that there exists N_2 such that for all $n \geq N_2$ $|F(p) - G_n(p)| = |\int_{-\infty}^{p} f - \int_{\infty}^{p+\frac{1}{n}} f| < \epsilon$. Then, letting $N_0 = max\{N_1, N_2\}$ we know that for all $n > N_0$, both $|\int_{-\infty}^{p} f - \int_{\infty}^{p-\frac{1}{n}} f|$ and $|\int_{-\infty}^{p} f - \int_{\infty}^{p+\frac{1}{n}} f|$ are less than ϵ. Letting $\delta < \frac{1}{N_0}$ we have:

$$|\int_{-\infty}^{p} f - \int_{-\infty}^{x} f| = |F(p) - F(x)| < \epsilon$$

whenever $|x - p| < \delta$. Then F is continuous at p. Since p was arbitrarily chosen, it follows that F is continuous. $\square$